Diagnostic Models of an Intelligent Tutor System for Teaching Skills to Solve Algebraic Equations

A. Kulik, A. Chukhray, M. Chukhray
National Aerospace University Kharkov, Ukraine

***Abstract*—In this paper one solution for teaching skills to solve n-power algebraic equation by Lobachevsky-Greffe-Dandelen method is described. Student's mistakes are discovered and classified. Based on signal-parametric approach to fault diagnosis in dynamic systems mathematical diagnostic models which allow detecting mistake classes by comparing student calculated results and system calculated results are created. Features of proposed diagnostic models application are presented. Intelligent tutor system is developed and used on "Automatic Control Theory" practical training by third year students of National Aerospace University.**

***Index Terms*—Intelligent tutor system, student mistake, diagnostic model**

I. INTRODUCTION

A key intellectual part of the computer tutor system, which incorporates the pedagogic mastery and experience of the best teachers, is its diagnostic service. It allows not only detecting a student's mistake, but also identifying the cause of the mistake in order to choose an educational sequence adapted for that student.

A quantity of scientific works is devoted to intelligent tutor system diagnostic service development. Part of them is based on the perturbated and reference models comparing approach [1,2,3,4,5]. Other part of works is based on Bayes approach [6,7]. Different implementations of intelligent tutor system are described. Such implementations as ALEKS, Carnegie Learning's Cognitive Tutors, Ms. Lindquist, ActiveMath [8] include diagnostic functions and teach to mathematical tasks solving. Diagnostic service in these systems is decomposed on two levels: one for specific student steps analyzing during concrete mathematical task solving and other for common student skills analyzing.

There are also opponents of diagnostic service in tutor systems. For instance, in work [9] authors hold the opinion that diagnostic aspect isn't necessary at all because educational process is divided on small stages and preventing from mistakes accumulation instantaneous feedback is used. We don't hold such opinion by following reasons. First of all, intermediate results input leads to additional time consumption. Secondly, in some cases there are several correct calculation orders [10] and uniquely defined educational process decomposition isn't possible. Thirdly even on small stage there is a possibility to make serious mistake, for instance, to approximate 3.445 to 3.5 or to 4.

This research is motivated by following reasons. In publications devoted to intelligent tutor diagnostic functions descriptions of students' mistakes refer usually to primitive mathematical actions. Descriptions of higher education institute students' professional mistakes, which appeared during practical tasks solving are almost absent. Mistake diagnosis mathematical tools are also weakly presented.

In this paper one solution for teaching skills to solve *n*-power algebraic equation by Lobachevsky-Greffe-Dandelen method is described. It is used on "Automatic Control Theory" practical training by third year students of National Aerospace University.

Lobachevsky-Greffe-Dandelen method [11] as a base to solve *n*-power algebraic equation was chosen because of its relatively simple calculation schema and importance for solving stability tasks due to possibility to determine both real and complex roots.

II. PROBLEM DEFINITION

1. To analyze n-power algebraic equation solutions written by students and on the basis of results discover and classify student's mistakes.
2. On the basis of the signal-parametric approach to fault diagnosis in dynamic systems [13] to create mathematical diagnostic models which allow detecting mistake classes by comparing student calculated results and system calculated results.
3. To develop intelligent tutor system for Lobachevsky-Greffe-Dandelen method learning which includes following main functions:

a) support student by guide which covered theoretical questions appeared during Lobachevsky-Greffe-Dandelen method learning;
b) task solution sequence control;
c) student knowledge and skills step-by-step diagnosis;
d) student knowledge and skills remediation by feedback introducing.

III. COMPUTER MODEL DESCRIPTION OF LOBACHEVSKY-GREFFE-DANDELEN METHOD

Let n, g be integer and positive numbers, h be integer nonnegative number, $b \in \{0,1,...,h\}$, $j \in \{0,1,...,n\}$, $l \in \{1,...,n\}, k \in \{1,2,...,g\}$ $v \in \{g-1, g\}$.

Let us consider the two simplest cases, when equation $a_0 x^n + a_1 x^{n-1} + ... + a_n = 0$, which coefficients are real numbers, has all real roots or only one pair of complex roots.

Let's introduce an auxiliary function $r\_f(x,h)$ to present by the rounding rules any real number x with a floating decimal point to within h digits after the point:

TABLE I.
CLASSES OF THE MISTAKES

Class of the mistakes	Number of mistakes	%
Mistake at imaginary part complex root calculation	14	13,3
Non-compliance with the condition of root squaring end	12	11,4
Mistakes connected with root squaring misunderstanding	11	10,5
Misspelling	10	9,5
Mistakes at doubled coefficients product calculation	8	7,6
Rough calculations	8	7,6
Unrecognized mistakes	6	5,7
Inverted formula for roots calculation	6	5,7
Lack of complex root existence conditions knowledge	5	4,8
Loss of sign at calculations	5	4,8
Redundant iteration	5	4,8
Incorrect coefficient exponentiation	3	2,9
Mistakes at 2^g –th root calculation	2	1,9
Displaced quotient calculation	2	1,9
Mistakes at exponent calculation	2	1,9
Others	6	5,7

$$r\_f(x,h) = (-1)^t \cdot (z_0 + z_1 \cdot 10^{-1} + z_2 \cdot 10^{-2} + +...+ z_h \cdot 10^{-h}) \cdot 10^p,$$ where $z_b \in \{0,1,...,9\}$, $t \in \{1,2\}$, p is a integer number, $(z_0 > 0) \oplus (\forall b\, z_b = 0)$.

Let's $A_{(0,j)} = a_j$. Then coefficients of reformed equations can be calculated by formula

$$A_{(k,j)} = r\_f(A^2{}_{(k-1,j)} + 2\sum_{s=1}^{j}(-1)^s A_{(k-1,j-s)} A_{(k-1,j+s)}, h),$$

where $A_{(k-1,c)} = 0$, if $(c < 0) \oplus (c > n)$.

Condition of coefficients calculation interrupting is

$$\forall j \forall v\, ((A_{(v,j)} > 0) \Rightarrow (\ r\_f(A_{(g,j)}, h) = = r\_f(A^2{}_{(g-1,j)}, h))).$$

Real roots calculation is performed by formula

$$x_l = \pm r\_f(\sqrt[2^g]{\frac{A_{(g,l)}}{A_{(g,l-1)}}}, h),$$

at that $\forall v (A_{(v,l)} > 0) \wedge (A_{(v,l-1)} > 0)$. Sign of real root is determined by substitution. At complex roots calculation $x_l = \alpha + i\beta$ and $x_{l+1} = \alpha - i\beta$ at first $\rho = |x_l| = |x_{l+1}|$, $\rho^2 = r\_f(\sqrt[2^g]{\frac{A_{(g,l+1)}}{A_{(g,l-1)}}}, h)$ is computed, where $\exists v (A_{(v,l)} < 0)$. Then $\alpha = r\_f(-\frac{1}{2}(\frac{a_1}{a_0} + x_1 + ... + x_{l-1} + x_{l+2} + ... + x_n), h)$ and $\beta = r\_f(\sqrt{\rho^2 - \alpha^2}, h)$ are computed.

IV. EXPERIMENTAL RESULTS

To determine concrete problems which appear when students solve algebraic equations we conducted experiment. 37 third-year students who study on a speciality «Automatic control system» took part in it. 20 different 3-power equations were prepared. Every equation has either 3 real roots or 1 real and 2 complex roots. Every student was offered to solve 1 equation. Necessary accuracy of calculations was 4 significant positions.

To reveal a maximum quantity of mistakes during the analysis of student's works we simulated calculations of the student even after detection of mistakes. In that way 105 different mistakes were detected.

From the point of view of mistake appearance reasons all mistakes can be divided into two classes: misconceptions and procedural mistakes [2].

Concerning to place of appearing all mistakes can be divided into two classes: general (for example, rounding mistakes, misspelling and so on) and specific for concrete place (for example, lack of complex root existence conditions knowledge)

Below discovered classes are presented ordered by number of mistakes appeared in our experiment (Table1).

Let's uncover essence of presented classes.

"Mistake at imaginary part complex root calculation" class describes situation when student forgets that absolute value of complex roots is directly calculated squared and thus substitutes ρ^4 instead of ρ^2 at β computing.

"Non-compliance with the condition of root squaring end" class is connected with ignorance of fact that not all significant positions of positive coefficients coincided with significant positions of squared coefficients received on previous step.

"Misspelling" class includes such typical spelling or input errors as single transcription (4 times), symbol deletion (3 times), symbol addition (once), adjacent symbols transposition (once), and multiple transcriptions (once).

"Mistakes connected with root squaring misunderstanding" are 2g-th root computing instead of 2^g-th root computing (8 times) and equation root calculation without computing of radical (3 times).

"Mistakes at doubled coefficients product calculation" class includes such mistakes as losing one of multipliers (4 times), using current left coefficient instead of top left (once), using initial coefficient (once), using current top coefficient instead of top left (once) and using current coefficients instead of top (once).

"Rough calculations" covers rounding mistakes (5 times) and ignorance of computing accuracy requirements (3 times).

Mistakes in "Inverted formula for roots calculation" class are caused by initial coefficient mirroring.

"Lack of complex root existence conditions knowledge" class is connected with complex root calculation when existence condition (coefficients sign interchanging in one of the columns) isn't true.

"Loss of a sign at calculation" is one of the most common classes of mistakes [12].

"Redundant iteration" class describes situation when student continues calculation when condition of interrupting is true.

"Incorrect coefficient exponentiation" class means raising to the fourth power instead of squaring (3 times).

"Mistakes at 2^g –th root calculation" class describes situation when student calculates 2^{g+1}-th root (2 times).

"Displaced quotient calculation" class appeared 2 times at ρ^2 calculation when student took $\frac{A_{(g,3)}}{A_{(g,1)}}$ instead of $\frac{A_{(g,2)}}{A_{(g,0)}}$.

"Mistakes at exponent calculation" class describes situations when significand is correct but exponent isn't.

"Others" class include such mistakes as incorrect power dividing ($\frac{10^{28}}{10^{14}} = 10^2$ (once)), β is calculating by formula for ρ^2 (once), incorrect sign determining of real root (once), incorrect sequence of operations ($(-10)^2 = -100$ (once)), lack of main algebra theorem knowledge (calculations only 2 roots instead of 3 (once)), taking 2^g-th root from one coefficient but not from quotient (once).

Maximum amount of mistakes made by one student was 6 (3 times). For instance, one student at first raised to the fourth power instead of squaring then misspelled one digit, later used current left coefficient instead of top left, then used inverted formula, later showed his lack of complex root existence conditions knowledge and at the end made mistake at imaginary part complex root calculation.

V. DIAGNOSTIC MODELS

Diagnostic model (DM) is a mathematical model which connects mistake with its symptom and allows solving of inverse problem [13, 14].

Let's introduce table of symbols: $\tilde{x}$ is a value calculated by student, $\hat{x}$ is a reference value calculated by tutor program, $m(r\_f(x,h)) = z_0 + z_1 \cdot 10^{-1} + z_2 \cdot 10^{-2} + ... + z_h \cdot 10^{-h}$ and $ex(r\_f(x,h)) = p$ are two auxiliary functions.

Then DM for mistake detecting is $r\_f(\tilde{x},h) \neq r\_f(\hat{x},h)$.

After mistake detecting we should identify its cause. So we use DMs for class finding. For instance, DM for finding "Mistake at imaginary part complex root calculation" class is $\tilde{\beta} = r\_f(\sqrt{\hat{\rho}^4 - \hat{\alpha}^2}, h)$.

DM for finding "Non-compliance with the condition of root squaring end" class is

$(\tilde{z}_0 = \hat{z}_0) \wedge (\tilde{p} = \hat{p}) \wedge (\exists s \in \{1,2,...,h\}\, \tilde{z}_s \neq \hat{z}_s)$.

DMs for "Mistakes connected with root squaring misunderstanding" are defined as follows.

$$\tilde{x}_l = \pm r\_f\left(\sqrt[2g]{\frac{\hat{A}_{(g,l)}}{\hat{A}_{(g,l-1)}}}, h\right),\ \tilde{\rho}^2 = r\_f\left(\sqrt[2g]{\frac{\hat{A}_{(g,l+1)}}{\hat{A}_{(g,l-1)}}}, h\right)$$

describe 2g-th root computing instead of 2^g-th root computing.

$$\tilde{x}_l = \pm r\_f\left(\frac{\hat{A}_{(g,l)}}{\hat{A}_{(g,l-1)}}, h\right),\quad \tilde{\rho}^2 = r\_f\left(\frac{\hat{A}_{(g,l+1)}}{\hat{A}_{(g,l-1)}}, h\right)$$ are defined for situation when roots or $\tilde{\rho}^2$ are calculated without computing of radical.

DMs for "Mistakes at doubled coefficients product calculation" class are:

$\tilde{A}_{(k,j)} = r\_f(\hat{A}^2{}_{(k-1,j)} + \sum_{s=1}^{j}(-1)^s \hat{A}_{(k-1,j-s)}\hat{A}_{(k-1,j+s)}, h)$ – loosing of 2;

$\tilde{A}_{(k,j)} = r\_f(\hat{A}^2{}_{(k-1,j)} + 2\sum_{s=1}^{j}(-1)^s \hat{A}_{(k-1,j-s)}, h)$ – loosing of right multiplier;

$\tilde{A}_{(k,j)} = r\_f(\hat{A}^2{}_{(k-1,j)} + 2\sum_{s=1}^{j}(-1)^s \hat{A}_{(k,j-s)}\hat{A}_{(k-1,j+s)}, h)$ – using current left coefficient instead of top left one.

DMs for "Rough calculations" class are defined by two following models.

$(r\_f(\tilde{x},h) - r\_f(\hat{x},h) = -1 \cdot 10^{ex(r\_f(\hat{x},h))-h})\wedge(\hat{z}_{h+1} \geq 5)$

serves for finding mistakes in rounding, where $\hat{z}_{h+1}$ is a stored reference (h+1) significant position. For instance, rounding mistake was made when student rounds 1,4445 to 1,444.

DM for finding mistakes connected with ignorance of computing accuracy requirements is defined as follows

$r\_f(\tilde{x},h) = r\_f(\hat{x},b),\ 0 \leq b < h$.

Using $1.6 \cdot 10^7$ instead of $1.631 \cdot 10^7$ can be seen as an example of such mistake.

To find single transcription we use DM which defined as $\exists b\,(\tilde{z}_b \neq \hat{z}_b) \wedge (\forall s \in \{0,1,...,h\} - \{b\}\, \tilde{z}_s = \hat{z}_s)$.

Adjacent symbols transposition can be found by using

$$\exists s \in \{0,1,...,h-1\}\,(\tilde{z}_s \neq \hat{z}_{s+1}) \wedge (\tilde{z}_{s+1} \neq \hat{z}_s) \wedge$$
$$\wedge (\forall w \in \{0,1,...,h\} - \{s, s+1\}\,\tilde{z}_w = \hat{z}_w)\,.$$

For finding all of “Misspelling” mistakes similar strings detecting methods [16] can be used.

DM for “Inverted formula using for roots calculation” class is

$$\tilde{x}_l = \pm r\_f(\sqrt[2^g]{\frac{\hat{A}_{(g,l-1)}}{\hat{A}_{(g,l)}}}, h)\,.$$

DM for “Lack of complex root existence conditions knowledge” class is defined as

$$(I(\tilde{x}_l) \neq 0) \wedge (\forall j \forall v (\hat{A}_{(v,j)} > 0)\,,$$

where $I(x_l)$ is an imaginary part of x_l root.

We define “Redundant iteration” as

$$\forall j \forall s \in \{g-2, g-1\}\,((\hat{A}_{(s,j)} > 0) \Rightarrow$$
$$\Rightarrow (\ r\_f(\hat{A}_{(g-1,j)}, h) = r\_f(\hat{A}^2{}_{(g-2,j)}, h)))\,.$$

$$x_l = \pm r\_f(\sqrt[2^g]{\frac{m(r\_f(\hat{A}_{(g,l)}, h))}{m(r\_f(\hat{A}_{(g,l-1)}, h))} \cdot 10^{\frac{ex(r\_f(\hat{A}_{(g,l)},h))}{ex(r\_f(\hat{A}_{(g,l-1)},h))}}}, h)$$

serves for finding of incorrect power dividing mistakes.

One of the possible scenarios of obtained DM application is presented in Fig 1 by the example of student’s coefficient $A_{(b,j)}$ calculation.

If student makes mistake the program will inform that mistake have been made, analyze mistake and then allow student to correct it. If mistake is made repeatedly the program again will inform that mistake have been made, analyze mistake and offer to repeat operations by steps. And only if student makes mistake on i-step the program will produce diagnostic message about mistake with specifying its class.

Such approach is agreed with a principle that student should at first work on mistakes without assistance. Thus the diagnostic message with a hint about the cause of the mistake is not given immediately. Instead of it, the opportunity to find and correct a mistake is given to the student. At the same time results of the mistakes analysis which is performed by the program on each step are stored in the student model.

All considered diagnostic models are created with taking into account only single student mistake, though students sometimes make also multiple mistakes.

The arguments for such assumption are.

1. Probability of multiple mistakes occurring is much less than single mistake occurring probability because $p_{er} < p_{er} \cdot p_{er} \cdot ... \cdot p_{er}$, where p_{er} – probability of single mistake occurring.

2. Even for not too complicated formulas when we take into consideration multiple mistakes huge amount of alternate solutions are appeared. It complicates the diagnostic service significantly [10].

3. If the diagnosis can’t be determined the program will ask student to repeat calculation by steps [2] or execute diagnostics conversationally with student [15].

Let’s note that several diagnoses may be inferred, for instance rounding from 1.3005 to 1.3 can be interpreted either as rounding mistake or as ignorance of computing accuracy requirements. In this case diagnosis is stored in student model as d1 ⊕ d2 and can be defined more exactly by additional questions to student.

We have implemented in Delphi 6.0 first version of such intelligent tutor system screenshots of which are presented in Fig. 2.

This system includes all defined above diagnostic models. When student misconception is found system returns him to the theory learning.

It is used on “Automatic Control Theory” practical training by third year students of our university. As results of its using we can note following. First of all even backward and lazy students work in tutor system with pleasure. Secondly every student correctly solves his task without teacher’s assistance.

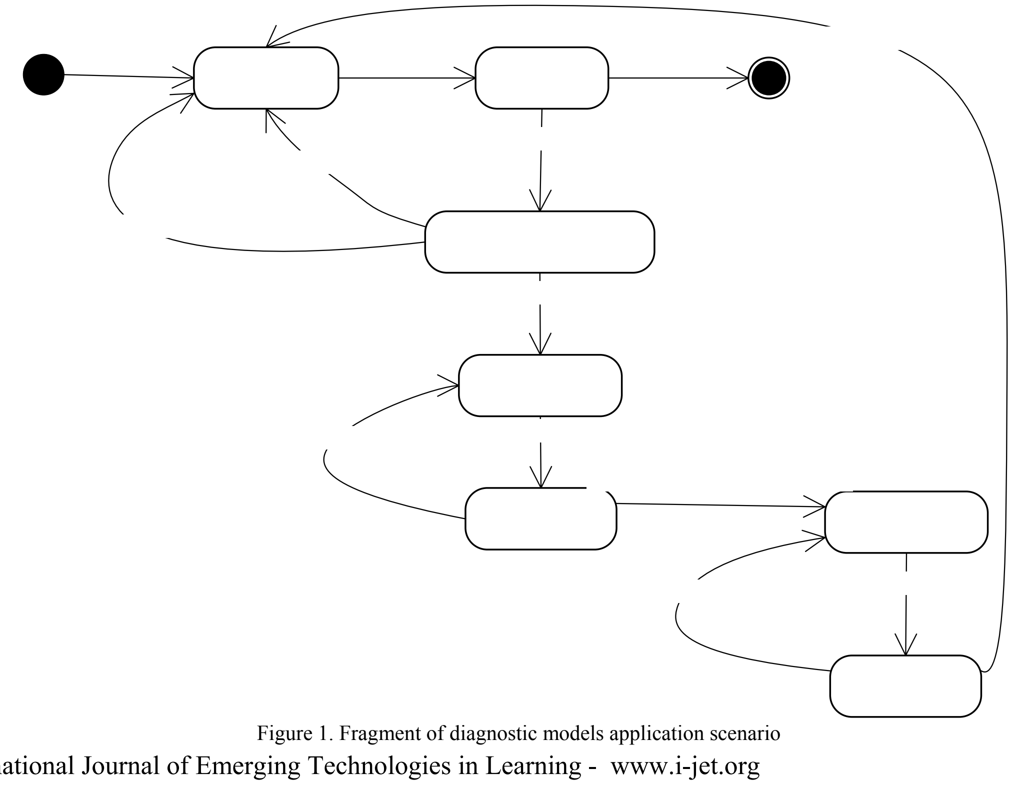

Figure 1. Fragment of diagnostic models application scenario

Figure 2. Intelligent tutor system screenshots

VI. Conclusions

Creating of diagnostic services is one of the central problems in intelligent tutor systems development. The main contributions of present paper are first of all the results of student's mistakes experimental research and secondly based on these experimental results proposed diagnostic models for student mistakes classes finding.

We are going to implement diagnostic models in interpreted language and to store it in database. Such approach will give potentialities to diagnostic models addition/modifying without any changes in program shell. Moreover program shell in mode of self-learning [1] will be able to generate new diagnostic models, store it in database and then interpret it. For instance, part of proposed models connected with missing of operations can be obtained automatically from reference model. We are also going to create diagnostic models for common student skills analyzing, such as estimating of attentiveness, ability to learning, ability to work on student's own and others.

References

[1]. Self, J.: Formal approaches to student modeling // Tech. Report AI-59, *Lancaster University*, 1991, http://citeseer.ist.psu.edu/self94formal.html

[2]. Stern, M.; Beck, J.; Woolf, B.: Adaptation of problem presentation and feedback in an intelligent mathematics tutor. // *In Proceedings of Intelligent Tutoring Systems*, 1996, pp. 605-613. http://citeseer.ist.psu.edu/stern96adaptation.html

[3]. Woo, C.: Instructional Planning in an Intelligent Tutoring System: Combining Global Lesson Plans with Local Discourse Control // Ph. D. Dissertation, *Illinois Institute of Technology*, 1991. http://citeseer.ist.psu.edu/woo91instructional.html

[4]. Liong Yu Tu; Wen-Lian Hsu; Shih-Hung Wu.: A Cognitive Student Model - An Ontological Approach. // *International Conference on Computers in Education, ICCE 2002*, December 3-6, 2002, Auckland, New Zealand. IEEE Computer Society, 2002, Volume 1, pp. 111-112. http://citeseer.ist.psu.edu/711614.html

[5]. Ragnemalm, E. L.: Student diagnosis in practice; Bridging a gap. // *User Modeling and User Adapted Interaction*, 5(2):93--116, 1996. http://citeseer.ist.psu.edu/ragnemalm95student.html

[6]. Nkambou, R.; Tchetagni, J.: Diagnosing Student Errors in E-Learning Environment Using MPE Theory // *In Proceedings of the International Conference on Web-Based Education*, 2004, pp. 249-254.

[7]. Millán E.; Pérez-de-la-Cruz, J.: Bayesian Diagnostic Algorithm for Student Modeling and its Evaluation.// *User Model. User-Adapt. Interact*. 12(2-3), 2002, pp. 281-330.

[8]. Shute, V.; Underwood, J.: Diagnostic Assessment in Math Problem Solving Educational testing service, *Princeton, NJ*, 2005, 7p.

[9]. Patel, A.; Kinshuk K.: Applied Artificial Intelligence for Teaching Numeric Topics in Engineering Disciplines. *Lecture Notes in Computer Science*, 1108, 1996, pp. 132-140. http://citeseer.ist.psu.edu/260351.html

[10]. Self, J.: Bypassing the intractable problem of student modeling. // *In Proceedings of Intelligent Tutoring Systems*, 1988, pp. 18-24. http://citeseer.ist.psu.edu/self90bypassing.html

[11]. Angot, A.: Compléments de mathématiques à l'usage des ingénieurs de l'électrotechnique et des télécommunications. *Paris: Centre National d'Etudes des Télécommunications*, 1957, 836 p.

[12]. Schechter, E.: The most common errors in undergraduate mathematics, 2006 http://www.math.vanderbilt.edu/~schectex/

[13]. Kulik A.: Fault Diagnosis in Dynamic Systems via Signal-Parametric Approach //*IFAC/IMACS Symp. Baden-Baden*, Sept. 10-13. – 1991, Vol.1, pp. 157-162.

[14]. Kulik, A.; Rasinkova, N.: Algorithmic Fault-Tolerance Support for a Gyroscopic Sensor Unit // *Proc. 3rd IFAC Symp. On Intelligent Autonomous Vehicles. – Madrid*., 1998, pp. 397-402.

[15]. Dimitrova, V.; Self, J.; Brna P.: Involving the Learner in Diagnosis - Potentials and Problems.// *Tutorial at Web Information Technologies: Research, Education and Commerce, Montpellier,France*,2000. http://citeseer.ist.psu.edu/dimitrova00involving.html

[16]. Kulik, A.; Chukhray, A.; Zavgorodniy, A.: Similar strings detecting methods // *In Proceedings of the East-West Fuzzy Colloquium 2005, Zittau, Germany*, 2005, pp. 183-191.

AUTHORS

Anatoly Kulik, Doctor of Technical Science, Professor, Laureate of Ukraine State Prize, head of Control Systems department, dean of Control Systems faculty in National Aerospace University, Kharkov, Ukraine (e-mail: kulik@d3.khai.edu).

Andrey Chukhray, Candidate of Technical Science, deputy head of Control Systems department, deputy dean of Control Systems faculty in National Aerospace University, Kharkov, Ukraine (e-mail: chukhray@d3.khai.edu).

Marina Chukhray, research assistant of Control Systems department in National Aerospace University, Kharkov, Ukraine (e-mail: mchuhray@mail.ru)

Manuscript received 16 October 2006. This work was supported by the President of Ukraine grant №GP/F11/0114/2006.