Huang, D., & Manouchehri, A. (2019). Online mathematics teacher education in the US: A status report. *Contemporary Issues in Technology and Teacher Education, 19*(2), 171-194.

Online Mathematics Teacher Education in the US: A Status Report

Dinglei Huang
University of Mount Union

Azita Manouchehri
The Ohio State University

The advancement of online technologies in recent years has increased the number of teacher learning opportunities offered in virtual environments. The development of the online medium for educational purposes has raised challenges for organizing and conducting professional development for teachers, especially relative to the ways subject matter disciplinary knowledge may be facilitated in such a medium. Using a survey instrument, data were collected from a national sample of mathematics teacher educators related to the content of their pedagogical practices in online mathematics teacher education. Results highlighted eight dimensions associated with mathematics teacher educators' decisions when designing and implementing online mathematics teacher professional development or academic courses. Knowledge required for mathematics teacher educators for each dimension of decisions is discussed.

With the recent advances in information communication technology, interest in the use of online distance learning has grown exponentially. In teacher education, using hybrid or purely online approaches to improve teachers' professional knowledge has also gained popularity in the past twenty years (Crockett, 2010).

Although exactly *when* online professional development was first introduced in the educational setting remains unclear, school systems across the world from as early as the mid-1900s have relied on technology to educate and train teachers, staff, and administrators (Crockett, 2010). The use of virtual technologies to provide distance teacher education depicts a leap forward in the history of educational technology, assuming teacher training can overcome temporal and spatial constraints. Yet, with regard to discussions about online education, practitioners and researchers have agreed that online education is not meant to replace the face-to-face system of education. Instead, it is meant to complement face-to-face education as an additional resource to provide extra learning opportunities (Keegan, 1993).

The online learning environment is now widely utilized in mathematics teacher education with the promise that it provides mathematics teachers with high-quality professional development opportunities, while accommodating for their busy schedules and geographic location (Dede et al., 2009; Whitehouse, Breit, McCloskey, Ketelhut, & Dede, 2006). Engelbrecht and Harding (2005) noted a number of potential benefits of online mathematics courses, including (a) the potential for student access to a wide range of educational resources online, (b) the potential for providing convenience and flexibility for training opportunities with regard to both time and place, (c) the potential for providing opportunities for a dynamic learning environment, (d) the potential for communication and collaboration beyond physical and regional boundaries, (e) the potential for independent learning, and (f) the potential for providing an environment in which students familiar with the digital world can integrate naturally.

With the increased interest in utilizing the online distance learning model in mathematics teacher education, mathematics teacher educators (MTEs) are involved in designing online courses and programs for mathematics teachers. Little is known about how mathematics teacher educators go about designing online programs for mathematics teachers, what difficulties they encountered, and what adjustments they made for delivering mathematics teacher learning sessions online. Yet, research relative to practices of online mathematics teacher educating has been limited.

We, therefore, conducted a national survey on mathematics teacher educators' practices of conducting online mathematics teacher education courses or professional development programs. The results of the analysis are shared here and identified a decision space including eight dimensions of decisions key to designing online mathematics teacher education courses.

Literature Review

To date, research on how the online environment affords teaching and learning has consistently been framed and theorized in terms of social interaction, which was originally derived from research on distance education. A mechanics view of interaction and dialogue was one of the earliest theoretical perspectives of distance learning (Gunawardena & McIsaac, 2004). Such a view draws on an industrial model of production where technology was used to reproduce teaching materials and learning occurred in individual self-study.

Researchers emphasized dialogue and interactions between the learner and the text in building knowledge (Holmberg, 1991). Students creating simulated conversation in the educational materials was encouraged as a method of learning from a distance. Though the emphasis of distance learning is still between the learner and the text, ideas of two-way reciprocal communication emerged in early distance education research.

A more recent theory of distance learning is credited to Moore's (1993) model of transactional distance, which "connotes the interplay among the environment, the individuals and the patterns of behaviors in a situation" (Boyd & Apps, 1980, p.5, cited in Moore, 1993). Moore highlighted the importance of the transactional view in distance education as the temporal and spatial separation of the teacher from the learner creates a special psychological and communication space. In this regard, Moore suggested that distance education must include multilayered dialogues between students and the teacher.

He identified three key interactive components in distance learning: (a) dialogue (i.e., interaction between learners and teachers); (b) structure (i.e., the design of the program or learning materials); and (c) autonomy (i.e., the degree of learners' self-directing in their own learning). He argued that learners exercise more autonomy when (a) greater structure and lower dialogue or (b) less structure and more dialogue are provided in the distance learning program. In other words, teachers could engage in more explicit interaction with learners through telecommunication technologies and open structured programs to increase learner autonomy or engage in tightly structured programs where maximum guidance, direction and advice are provided by course designers to increase learner autonomy.

Larkin and Jamieson-Proctor (2015) used Moore's transactional distance theory to design an online mathematics teacher education program. They reported that online distance learning did not necessarily encompass the reciprocal relationship between structure and dialogue. Rather, their learners had additional autonomy when more structure and more dialogue were provided.

In the mechanics view, the teacher's role was not considered as critical. Moore's theory suggested teachers' intervention would greatly reduce learners' autonomy. Moore's perspective has been challenged by those who espouse a constructivist view of teaching and learning. Constructivist-based theories of distance education place a greater emphasis on the role of community and teacher. They suggest that such a role has become even more prominent when the online environment is the dominant medium of learning for distance learning (Anderson, 2008; Garrison, Anderson, & Archer, 2001, 2010; Lehman & Conceição, 2010).

Anderson, Rourke, Garrison, and Archer (2001) in a study of teaching in the online environment theorized the online community of inquiry as a construct that integrates cognitive presence, social presence, and teaching presence. Among these three presences, one emphasizes the specific content development (cognitive); one focuses on the collaborative context and presence of real and functional human beings instead of machines (social); and the third accounts for a supportive or facilitative role for the above two (teaching).

Anderson and colleagues regarded teaching presence as the most important element (Anderson, 2008; Anderson et al., 2001). They posited that teaching in the online environment involves "devising and implementing activities to encourage discourse between and among students, between the teacher and the student, and between individual students, groups of students, and content resources" (Anderson, 2008, p. 345). In other words, the teacher's role in online teaching is to devise content-based discourse among online participants so communities of inquiry can be established.

The researchers emphasized that teaching in the virtual learning environment must go beyond moderating and that the online teacher must add subject-matter expertise through a variety of forms of instructional actions. They opposed a laissez-faire "guide-on-the-side" approach in online teaching but, rather, emphasized the provision of rigorous instruction. As such, the online teacher should support and contribute to the learning discourse community by "injecting comments, referring students to information resources, and organizing activities that allow the students to construct the content in their own minds and person contexts" (Anderson et al., 2001, p. 9).

Vertecchi (1993) proposed an organizational structure for characterizing the role of a teacher in the distance education system. He argued that unlike traditional education, where different functions of teaching (e.g., planning the teaching methods, preparing content, communicating the content to student, and assessing student learning results) are accomplished by a single teacher, different teaching functions are separated out and specialized by individual teachers in their specific roles:

- Designing: outlining and defining the learning goals and the audience of the education, and also selecting technological tools and communications systems;
- Development: preparing all the content materials needed; and
- Implementation: communicating with students about the material such as distributing the study materials, and grading assignments.

Vertecchi argued that the success of the online learning project depended on the logical design of the project and the internal coordination of each specialization part. Vertecchi further proposed that the role of the teacher in distance learning has become so specific that a single teacher can no longer play a defining part in one's learning.

Online Mathematics Teacher Education

Online mathematics teacher education concerns not only online teaching but also the complex issues of mathematics teachers' learning. A search of keywords, "virtual," "online," and "mathematics teacher" in major journals and relevant databases in mathematics education resulted in about 25 groups of mathematics teacher educators around the world who had published studies on online mathematics teacher education within the past 5 years. Collectively, these studies concerned the design of online courses or programs for in-service or prospective mathematics teachers.

Among the different types of online mathematics teacher education courses or programs studied, some addressed the design for an entire graduate study program, including multiple online courses. Some were about the design of a professional development program, and others were about the design of an online mathematics teacher educational course including methods courses and technological integration courses, science, technology, and engineering, as well as science, technology, engineering, and mathematics (STEM) integration courses, and so forth. Each online course or program has its own unique features and focus regarding the its design.

Most existing literature outlined optimal organizational structures of the online teacher education programs, such as module structure, participation structure, and types of technology used. Teacher collaboration, engagement, and autonomy are the themes that were generally addressed (Chieu & Herbst, 2016; Clay, Silverman, & Fischer, 2012; Hjalmarson, 2017; Kastberg, Lynch-Davis, & D'Ambrosio, 2014; Larkin & Jamieson-Proctor, 2015; Pape et al., 2015).

For instance, Pape et al. (2015) described a yearlong, asynchronous online teacher education program for mathematics teachers with a focus on mathematical knowledge for teaching Grades 3 to 5. Their program was organized around weeklong modules that included activities across 35 weeks. Each module consisted of an introduction, anticipatory activity, content and discussion, and classroom connections.

The activities were presented in video-recorded slideshow presentations. The teacher educators placed emphasis on social presence in the online environment where norms of participant-to-participant interactions were purposefully established. In addition, they emphasized cognitive presence in their online program, where they ensured that participants had multiple opportunities to do mathematics and engaged their colleagues in mathematical conversations.

Chieu and Herbst (2016) used video records of teaching to create a series of animations of cartoon characters in classroom situations to engage teachers in an asynchronous online forum discussion about teaching. *Social presence* was emphasized in their course design;

the online participants were encouraged to discuss teaching events in the animations and reflect on important teaching decisions that occurred during those events. Online conversations were the places where the instructors identified learning outcomes.

Along the lines of online conversations, Hjalmarson (2017) asserted that the online platform was not the prime reason for communication difficulties. Instead, communication difficulties arose when the teachers were forced to work with other teachers in the online platform. As an online course designer, Hjalmarson felt that understanding how to facilitate communication and conversations among the teachers with the use of online learning technology was critical.

Kastberg et al. (2014) reflecting on design and implementation of an asynchronous course on proportional reasoning for in-service teachers, emphasized the importance of establishing a relationship between the online teacher educators and the course participants. The authors said that their failure in getting to know their teachers beyond the first session restricted them in addressing the teachers' context-based needs. The authors suggested that constructivist listening was critical for teaching in the asynchronous environment. Ways of listening might include prompt response to the participants' discussion posts or willingness to be helped by the participants with technological issues.

Many online mathematics teacher education courses have their original version conducted in the traditional face-to-face environment. A couple of studies examined the redesign of their face-to-face courses. Larkin and Jamieson-Proctor (2015) redesigned their face-to-face mathematics education course for elementary preservice teachers for online delivery. The teacher educator organized the online course with a lecture component and a synchronous interactive classroom component, guided by a theory about course structure, course dialogue and student autonomy.

Course structure referred to the rigidity or flexibility of the course in response to individual students' learning needs; course dialogue referred to interactions between the instructor and the learners; and learner autonomy recognized the importance of the learner to determine their learning experiences. The theory suggests an inverse relationship between course structure and course dialogue, as well as learner autonomy in face-to-face courses; that is, the more structured the course design, the less dialogue and student autonomy.

After two rounds of implementation and modifications of the design, the authors found that structural elements, such as video lectures and dialogue fostered by synchronous virtual classrooms, did not necessarily work in opposition to each other in the online learning environment. In fact, their students' feedback indicated that the structure and dialogue elements enabled more learner autonomy and supported their development of the use of virtual manipulatives for better understanding.

Besides participation and organizational structures, Schwartz (2012) studied the differences of one mathematical task when implemented in the face-to-face environment versus when implemented in the online environment. The mathematical problem that the author used was a game called "I Can Count to 20 Before You Can!" – a task for a K-2 mathematics methods course. In the face-to-face class, the instructor first modeled the game, then students played with a partner to develop strategies for winning the game. The instructor encouraged the students to list conjectures on the board and then discussed the strategies in a class group format.

In the online course, the students were given written directions about the game via online assignment; they then worked in teams to figure out how to win the game. Students in the online class were required to post to the discussion board and provide meaningful comments to group members. Schwartz (2012) reported that the change in ways the task was implemented resulted in different learning outcomes. The online students were more detailed in their mathematical representations and systematic in their conjecture and thinking, while the face-to-face class had more opportunities to discuss the pedagogical implications of the game. After reflecting on the experience, the author said that the online environment could not model the same teaching methods as the face-to-face environment, because the learning outcomes were different due to the learning environment.

In summary, as in the research on online teaching, studies in mathematics teacher education were concerned about the issues of social interactions, teacher presence, and the community of learning in the online environment. These studies started discussions on a few pedagogical issues, such as organizing online discussion, structuring online interaction, and changing mathematical task learning outcomes caused by the online medium. These issues are critical, but they do not represent a bigger picture of what issues that may be involved in online mathematics teacher education. A status report of practices of online mathematics teacher educating is needed in order to provide a comprehensive understanding of online mathematics teacher educating so as to support the development of mathematics teacher educators.

Theoretical Framework

The literature suggests that teaching functions in the online learning environment share similarities with usual face-to-face teaching. Yet they are different, which potentially frames mathematics teacher educators' practices when they design and implement online education experiences for preservice or in-service mathematics teachers. The theoretical framework for this study draws from Boyd's (1993) distance learning theory on decision dimensions.

Boyd's (1993) theory is prescriptive in nature in that it aims to account for all stakeholders in distance learning. Boyd identified five vital functions for distance education institution – five discourse spaces that structure virtual distance education (Boyd referred it as "cyberspace"):

- System V discourse spaces: a constitutive discourse space to set the organization up; and a judiciary discourse space to deal with infringements of regulations.
- System IV discourse spaces: an anticipatory-intelligence discourse space to look outside and into the future.
- System III discourse spaces: a task allocation and monitoring system; a recruitment/marketing system to recruit students and staff, and market courseware; and a resource acquisition and waste disposal system.
- System II discourse spaces: the resource allocation, monitoring and balancing system.
- System I discourse spaces: instructional design and production systems; teaching broadcasting/publication distribution systems; learning-teaching conversation discourse-space systems; and learner support discourse sub-systems. (p. 246)

Boyd (1993) depicted how the system functions with all of these discourse spaces in distance education. All five systems account for the activities of the participants in system. The current study focused exclusively on System I discourse spaces, where instructional design and production is situated. Within System I, Boyd outlined eight descriptive and prescriptive dimensions of decisions essential to the instructional design and production. These eight dimensions include the following:

1. agreed goals (i.e., “what for”);
2. corresponding rules and mechanism that make sure goals are achieved (i.e., “how to control”);
3. subject matter (i.e., “what”)
4. corresponding metaphors and views that illustrate the content (i.e., “how to illustrate”);
5. choices of real and virtual spaces (i.e., “where”);
6. corresponding media used in the space (i.e., “through which”);
7. participants (i.e., “who”); and
8. corresponding sociostructure of the participants (i.e., “with whom”).

Since all eight dimensions were developed specifically for a distance education research or development project, they frame the design of the survey to capture the essential elements constituted in mathematics teacher educators’ practices of designing and implementing content-specific online teacher education.

Methodology

The purpose of the current study was to capture mathematics teacher educators’ (MTEs) practices relative to designing and conducting teacher education in the online learning environment. The development of the survey took approximately 6 months. During the preparation stage, we conducted five cognitive interviews to receive feedback on the content, clarity, and length of the survey. After multiple rounds of modifications, we collected a sample of pilot responses from 26 mathematics teacher educators in June 2016, which was utilized to finalize the survey instrument. For example, we changed some open-response questions on the pilot survey to multiple choice format based on the pilot results. The final survey was administered via Qualtrics, an online survey platform, in September 2016.

The following research questions guided the overall design of the survey:

1. What types of courses or professional development (PD) programs were provided for mathematics teachers online?
2. What were the activities that were conducted online? How did MTEs organize these activities in comparison to the face-to-face environment?
3. What difficulties did MTEs encounter when they designed and implemented online teacher education?
4. What adjustments did they make to the PD content design to accommodate the online environment?
5. How did MTEs view the efficacy of the online environment for providing mathematics teacher education?

The survey consisted of four sections. The first section captured general information about the participants’ professional backgrounds, including the types of institutions for which they worked, numbers of years of experience working with teachers, and years of experience conducting mathematics teacher education online. The questions were in multiple choice format.

The second part of the survey asked the participants to share information about an online mathematics teacher education course or a PD program that they had designed and taught. The participants were asked to identify the type of activities conducted online. For those who identified that they had conducted a similar course face-to-face, they were asked about the type of activities conducted face-to-face. Figure 1 shows a sample question. Participants were asked to identify the frequencies of the activities they used in their online sessions. These activity choices were developed from the pilot study.

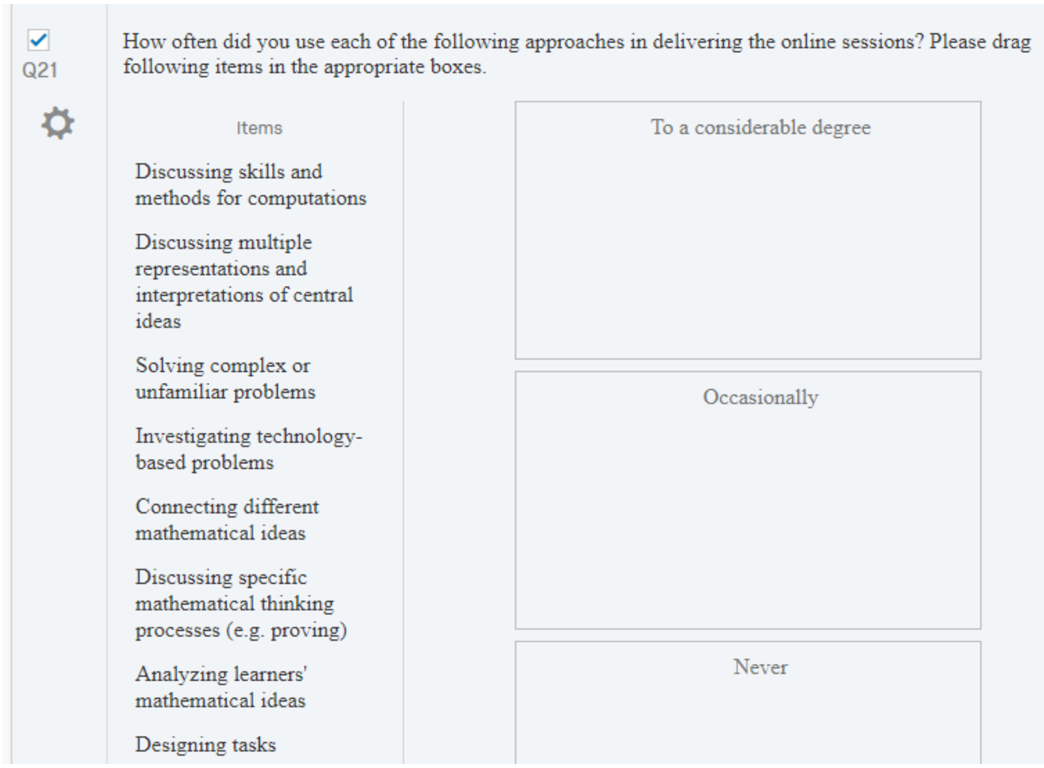


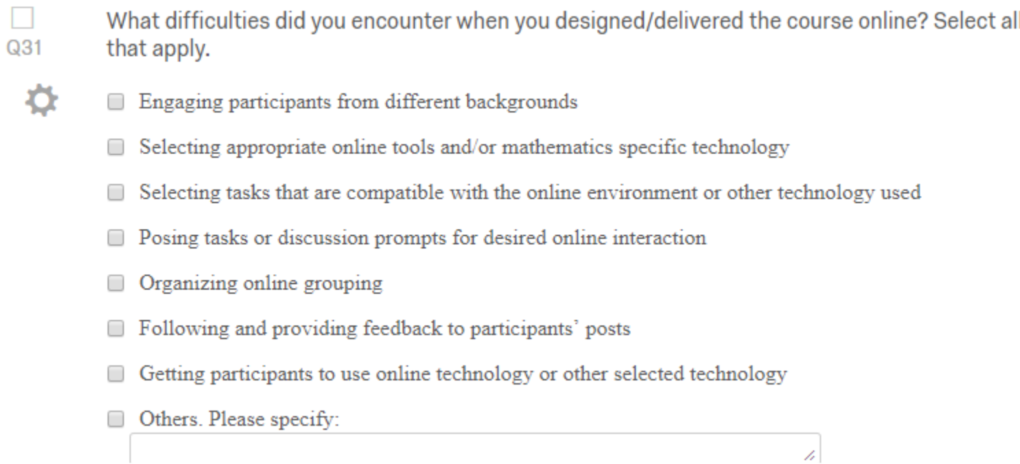
Figure 1. *Survey section II sample question screenshot: Activity conducted online. (See the appendix for text versions of sample questions*

The third section of the survey obtained information about how they designed the course and what adjustments they felt were necessary to be made when organizing the course or PD content activities for online delivery. They were also asked to report the type of difficulties they encountered when they designed and implemented the online course or PD (Figure 2).

The last section of the survey asked MTEs to share their perspectives on their experiences of conducting online mathematics teacher education sessions in comparison to a face-to-face delivery format. This last section provided evidence of how the online environment had impacted MTEs' practices (Figure 3).

Survey Participants

The online survey was administered to mathematics teacher educators in the United States. We collected more than 800 email addresses from a major national organization of which the majority of MTEs are members. We sent email soliciting consent to participate in the study to all potential candidates. Of that number, 114 mathematics teacher educators responded to the invitation and agreed to complete the survey. Among them, only 68 MTEs, who had conducted online mathematics teacher education courses with more than 50% of learning activities occurring online, were identified as qualified candidates for the study.

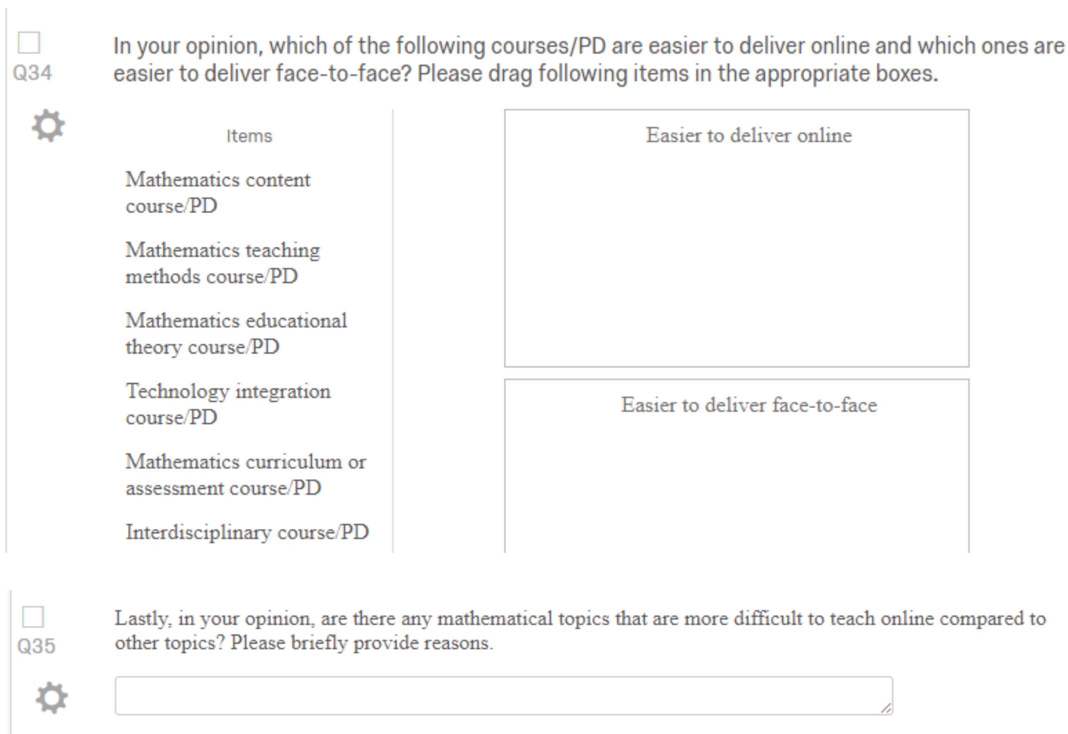


Q31

What difficulties did you encounter when you designed/delivered the course online? Select all that apply.

- Engaging participants from different backgrounds
- Selecting appropriate online tools and/or mathematics specific technology
- Selecting tasks that are compatible with the online environment or other technology used
- Posing tasks or discussion prompts for desired online interaction
- Organizing online grouping
- Following and providing feedback to participants' posts
- Getting participants to use online technology or other selected technology
- Others. Please specify:

Figure 2. *Survey section III sample question screenshot: Difficulties encountered.*



Q34

In your opinion, which of the following courses/PD are easier to deliver online and which ones are easier to deliver face-to-face? Please drag following items in the appropriate boxes.

Items

- Mathematics content course/PD
- Mathematics teaching methods course/PD
- Mathematics educational theory course/PD
- Technology integration course/PD
- Mathematics curriculum or assessment course/PD
- Interdisciplinary course/PD

Easier to deliver online

Easier to deliver face-to-face

Q35

Lastly, in your opinion, are there any mathematical topics that are more difficult to teach online compared to other topics? Please briefly provide reasons.

Figure 3. *Survey section IV sample questions screenshot: MTEs' opinions about online vs. face-to-face.*

The majority of the qualified candidates were faculty members working in either research-focused institutions or teaching-focused institutions. Others included five participants who identified themselves as graduate students, two as education consultants, and four as mathematics or educational specialists. Thirty-eight percent of the participants had more than 15 years of experience working with mathematics teachers, and only 16% of the participants had less than 5 years of experience. The majority of the participants were new to online teacher education, with more than two thirds of the sample stating that it was their first time developing online mathematics teacher education courses or PD programs within the past 5 years.

Data Analysis

The analysis of the survey was divided into four parts. In Part I, we used descriptive statistics to provide background information on the survey participants' backgrounds and their preferred logic maps when designing online programs.

Part II intended to capture the characteristics of online mathematics teacher education courses or PD programs that were reported by the survey participants. We first categorized the types of online mathematics teacher education courses according to MTEs' descriptions. We then used descriptive statistics to report the characteristics of the online courses and PDs in each category. Such characteristics included (a) mathematics content strands that were offered; (b) the targeted course/PD participants; (c) grade bands; (d) online delivery formats; (e) the foci of the course/PD; (f) MTEs' choice of mathematics-related activities; (g) MTEs' choice of pedagogical related activities; and (h) methods of participant online interaction.

In considering information for Part III, we first analyzed the multiple-choice options concerning ways the MTEs designed the online teacher education sessions using descriptive statistics. We then used the *grounded theory approach* (Corbin & Strauss, 2007) to analyze the open-response items in an effort to categorize the kinds of considerations and adjustments that the MTEs had made for the online delivery. All survey participants' responses to each question were recorded.

Clusters that emerged were labeled. We generated six categories of codes, which were adjustments about (a) tasks and activities design, (b) technological tools, (c) online platforms, (d) content illustration methods, (e) control system, and (f) groupings. Then, we coded terms and phrases using the six codes for all responses. Frequencies of the codes were calculated and are reported in the following section.

Findings

The reported online mathematics teacher education programs ranged from traditional college education courses to Massive Open Online Courses (MOOCs) with over 200 participants. A third of the courses were hybrid, meaning they involved some level of in-person sessions. About half of the online programs met in a synchronous-only format, and the other half of the programs offered all online activities asynchronously; even so, all of these online courses or PD programs had more than 50% of learning activities occur online. About two thirds of the reported online mathematics teacher education programs were PD for practicing teachers, while 19% of the programs were university courses for prospective teachers. The remaining 19% of the courses were courses for both prospective and practicing teachers.

A frequency diagram (Figure 4) was produced to summarize the foci. The vertical axis represents the topics, and the horizontal axis depicts the frequency of the counts of the participants who identified the topics. According to the figure, "mathematics and teaching methods," "mathematics and learners' mathematical thinking," and "teachers' mathematical knowledge" were the three top focusing themes. Other topics, such as curriculum, research and theory, and mathematics teacher leadership, were also reported by the participants.

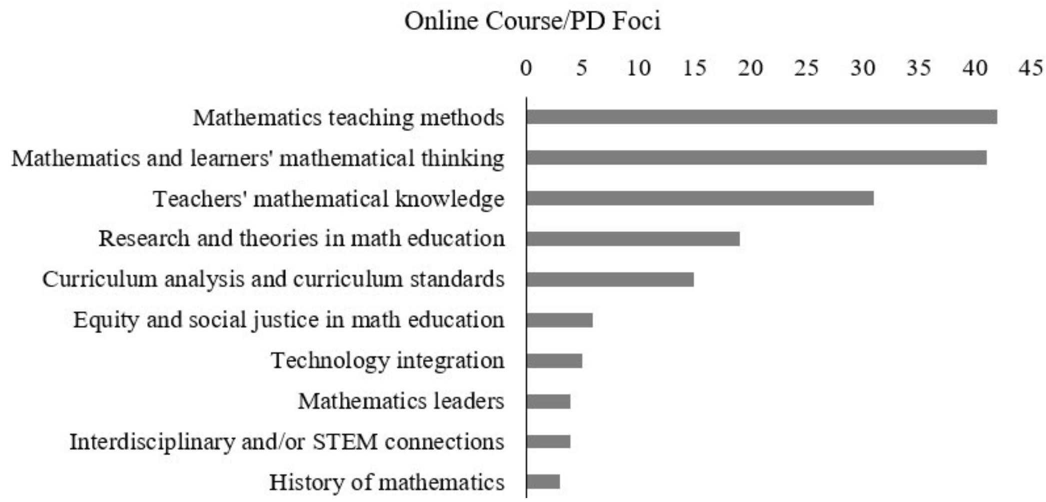


Figure 4. *Foci of online mathematics teacher educational programs.*

In terms of mathematical content areas, the online programs were not exclusive to a single mathematical strand. In fact, only six MTEs reported that their online program focused on a particular topic such as geometry, statistics, or discrete mathematics. These programs targeted middle school curriculum or higher levels of mathematics. Others reported that multiple content strands were involved in their programs. Some of these programs were designed for early childhood or elementary school teachers, and some focused on mathematical modeling, problem solving, or mathematical reasoning, where by the nature of their topics, multiple mathematical concepts could be discussed.

One MTE said that mathematical content was the “backdrop” for their discussions of teaching methods, curriculum, and so forth. A few MTEs indicated that mathematical content areas did not apply to their online programs, where discussions of history, theory, or research methods were the targeted content. In general, the survey results indicated that a variety of professional learning opportunities were offered online for mathematics teachers. Furthermore, mathematics teacher educators’ responsibilities did only cover not teachers’ development in mathematics teaching and learning, but also teachers’ development as professionals, including development of knowledge of theory and research and capabilities of working with other professionals.

Table 1 ranks the popularity of the activities that mathematics teacher educators used in their online courses or PD programs. The ranking was derived from the number of counts where the participants indicated devoting a considerable amount of time to the activities. As shown in the table, “discussing multiple representations,” “analyzing teaching practices,” “connecting different mathematical ideas,” “analyzing learners’ mathematical ideas,” and “discussing specific mathematical thinking process” were the top five popular activities in the online teacher education sessions. The total counts exceeded 25 for these five categories, and they were not particularly different. The results are consistent with the results of content foci where mathematics teaching methods, learners’ mathematics, and teachers' mathematical knowledge were indicated to be the most common in the online teacher education programs.

Table 1
Ranks of Popularity of Online Course/PD Activities

Rank	Online Course/PD Activities	Counts
1	Discussing multiple representations and interpretations of central ideas	30
2	Analyzing teaching practices (e.g. questioning, feedback)	29
2	Connecting different mathematical ideas	29
4	Analyzing learners' mathematical ideas	27
5	Discussing specific mathematical thinking processes (e.g. proving)	26
6	Designing tasks	20
7	Solving complex or unfamiliar problems	19
8	Assessing student learning	18
9	Discussing research in mathematics education	15
10	Reviewing curriculum materials	9
10	Discussing skills and methods for computations	9
12	Investigating technology-based problems	5
13	Discussing social justice issues	4

Thirteen participants responded that they had conducted a similar face-to-face course prior to the online course. Table 2 shows the rank of popularity of activities reported by these participants when they taught the similar course or PD sessions in the face-to-face environment. Similar to Table 1, the ranking was derived from the number of counts where the participants indicated devoting a considerable amount of time to the activities. The top three activities identified were "discussing multiple representations," "connecting different mathematical ideas," "analyzing teaching practices," "discussing specific mathematical thinking process," and "analyzing learners' mathematical ideas," which are the same as indicated in the online courses despite the number of participants who reported this question was significantly lower.

Table 3 provides a simple analysis of how these ranks changed while the similar courses/PD sessions were taught face-to-face. As shown in the last column of the table, the changes of ranks were within two places and mostly one place. In other words, the changes in popular activities conducted in the face-to-face environment versus in the online environment were small, given a smaller number of participants reported in the face-to-face activities question (because a smaller number of participants conducted both online and face-to-face versions of the same or similar courses).

As discussed in the Table 1 analysis, the popularity of activity choice is highly consistent with the results of content foci, where mathematics teaching methods, learners' mathematics, and teachers' mathematical knowledge were indicated to be the most common in the online mathematics teacher education programs. Based on the comparisons, one may conjecture that MTEs' choice of activities were based upon the course objectives, while the online environment did not have much impact on the choice of activities.

These data are aggregate based on anonymous responses, so we were not able to separate individuals to compare and contrast their choices. However, despite of the differences in number of responses collected for Table 1 and Table 2, the similarity in the results was astounding.

Table 2
Ranks of Popularity of Face-to-Face Course/PD Activities

Rank	Face-to-Face Course/PD Activities	Counts
1	Discussing multiple representations and interpretations of central ideas	12
1	Connecting different mathematical ideas	12
3	Analyzing learners' mathematical ideas	11
4	Discussing specific mathematical thinking processes (e.g. proving)	9
4	Analyzing teaching practices (e.g. questioning, feedback)	9
6	Assessing student learning	8
7	Solving complex or unfamiliar problems	7
8	Designing tasks	5
9	Discussing research in mathematics education	3
9	Discussing skills and methods for computations	3
11	Reviewing curriculum materials	2
11	Investigating technology-based problems	2
13	Discussing social justice issues	1

Table 3
Changes of Activity Ranks in Online Courses Versus Face-to-Face Courses

Activities	Online Rank	Face-to-Face Rank	Change
Discussing multiple representations and interpretations of central ideas	1	1	Same
Analyzing teaching practices (e.g. questioning, feedback)	2	3	↓1
Connecting different mathematical ideas	2	1	↑1
Analyzing learners' mathematical ideas	4	3	↑1
Discussing specific mathematical thinking processes (e.g. proving)	5	4	↑1
Designing tasks	6	8	↓2
Solving complex or unfamiliar problems	7	7	Same
Assessing student learning	8	6	↑2
Discussing research in mathematics education	9	9	Same
Reviewing curriculum materials	10	11	↓1
Discussing skills and methods for computations	10	9	↑1
Investigating technology-based problems	12	11	↑1
Discussing social justice issues	13	13	Same

Face-to-Face Versus Online

To investigate whether MTEs would select different activities for online delivery compared to face-to-face delivery, the survey participants were asked to indicate whether they conducted a similar PD or taught in a similar course prior to the online delivery. Thirteen MTEs indicated they had conducted a similar course or PD prior to the online format. The survey asked these MTEs to rate the extent to which they used the list of Table 1 activities in the face-to-face environment. Seven of them rated differently in some of the activities for the frequencies of use in the face-to-face environment. Due to the fact that the number of participants was small, no general pattern was found with regard to what activities were more frequently or less frequently used in the face-to-face rather than in the online environment.

Yet, all participating MTEs, regardless of whether they had taught a similar course or PD in the face-to-face environment, expressed that certain courses were much easier to implement in the face-to-face environment than in the online environment. The bar graph in Figure 5 summarizes the MTEs' perspectives on the kind of course that they believed to be easier to deliver online.

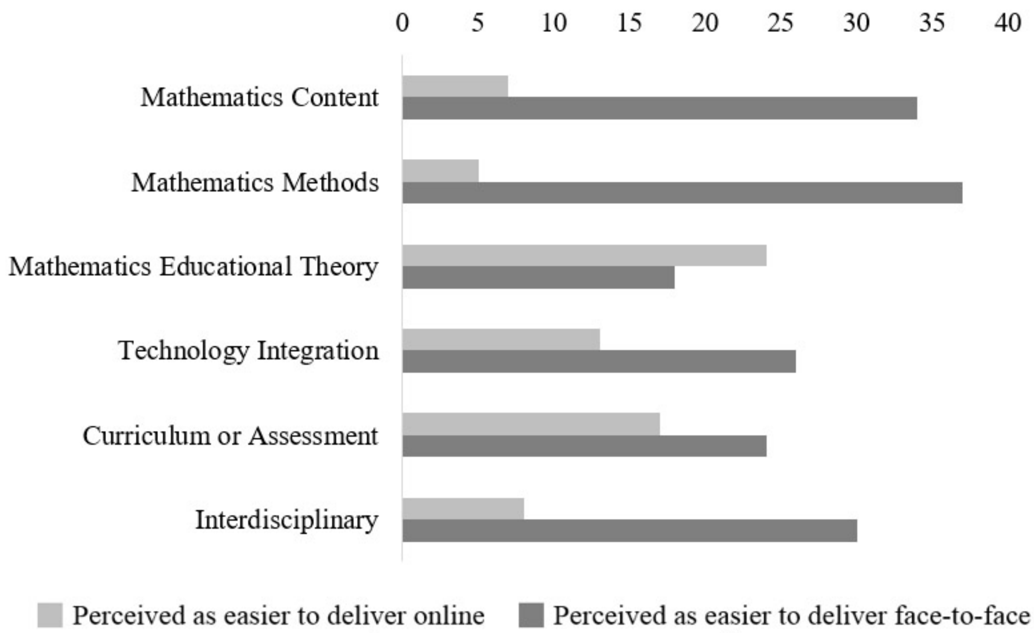


Figure 5. *MTEs' opinions on courses easier to deliver online versus face-to-face.*

Certain courses had a much higher number of votes for face-to-face delivery than online delivery: mathematics content courses/PD, mathematics teaching methods courses/PD, interdisciplinary courses/PD, or technology integration courses/PD. Mathematics content weighs heavily in the delivery in these courses or PD programs. Courses like mathematics educational theory, curriculum, or assessment courses, which were perceived as easier to deliver online, had a weaker preference for face-to-face delivery over online delivery. This phenomenon seems to indicate that intense mathematics content courses were more challenging to deliver in the online environment. This result could be the issue with the nature of having mathematics or mathematics-related discussions in these courses or PD sessions. Also, despite certain courses or PD sessions being preferred to be delivered face to face, a small number of MTEs still preferred online delivery. Ultimately, there was no absolute preferable method of delivery.

Difficulties Encountered

The survey had a multiple-choice question that asked the participants to identify difficulties they encountered when implementing the online teacher education program. Table 2 provides the percentage of selections. As the summary indicates, the first two difficulties identified were about task design and task selection for online delivery. More precisely, the difficulties were about designing tasks that could generate online interaction and tasks that were compatible with online technology. Some MTEs in the "others" selection expressed that modifying content that had been used in the face-to-face environment into a web-based presentation was a key challenge. These difficulties were both technological (as they were specifically related to the online environment) and pedagogical (as they were about task design).

Table 4
Difficulties When Designing and Implementing Online Courses

Difficulties Encountered	%
Posing tasks or discussion prompts for desired online interaction	57%
Selecting tasks that are compatible with the online environment or other technology used	43%
Following and providing feedback to participants' posts	41%
Organizing online grouping	32%
Engaging participants from different backgrounds	25%
Selecting appropriate online tools and/or mathematics specific technology	25%
Getting participants to use online technology or other selected technology	23%
Others.	14%

The third and fourth most frequently identified difficulties were about facilitating social interaction in the online environment, as they were following posts, providing feedback, or organizing grouping. Again, these difficulties were also pedagogical and technological. Some of the challenges were not listed in the survey but expressed in the "others" selection, including knowing if participants were actively and consistently engaged.

Additionally, the time needed to plan online sessions was another issue. One MTE claimed to spend "at least 6 times as much work as face-to-face teaching." Indeed, conducting mathematics teacher education in the online environment exerted new pedagogical challenges for MTEs. They had to adjust their goals and content activities to the online environment that was different from the face-to-face environment that they had used previously.

Adjustments Made

An open-response question asked the participants to identify and rate the importance of the revision they had made to the PD content and content organization. Fifty-seven participants responded to this question. Using the grounded theory approach (Corbin & Strauss, 2007), six themes of adjustments emerged: (a) tasks and activities design, (b) technological tools, (c) online platforms, (d) content illustration methods, (e) instructional methods, and (f) groupings. The total counts of these types were calculated and examples were listed as illustrated in Table 5.

Nearly 72% of the responses identified that task design or activity design were the key adjustments they made for online delivery. Task design was about the choice of task types – what type of tasks are feasible or not feasible in the online platform. As shown in the example column, one MTE chose product-oriented type of tasks for students to produce, and each product utilized various online technologies such as video, Wiki, or online discussion board.

Table 5
Types of Adjustments Made by MTEs

Adjustments	Count	Examples
Task/activity design	41	"Most activities had a product for students to produce - a short reflection, a video, an outline, discussion board participation, a Wiki, etc." "How to engage participants in authoring representations (rather than just consume)" "Had to abandon all the face-to-face activities like game playing, modeling group instruction, and debates." "My colleagues and I developed several protocols to follow (both synchronously and asynchronously) to help facilitate conversation - deep conversation about teaching - between participants."
Technological tools	14	"I substituted technology tools for a lot of the geometry explorations which I usually do with pencil and paper." "I used many more online resources, like applets and video, that students were expected to investigate ahead of time."
Online platforms	12	"I thought about the organization of the course and which environments to use in each year also - to support teachers' discussion, collaboration, and dissemination of what they learned and created."
Content organization	9	"Content was organized into discrete modules, each with a variety of short activities." "I structured the content in the form of the performance-based assignments for the course that paralleled what they were learning about in each session."
Instructional methods	4	"included more lectures"
Grouping	1	"how I grouped students"

Another MTE chose "authoring representations" types of tasks, which are similar to product-oriented tasks where students were assigned to produce rather than consume during online learning. One MTE mentioned that activities like "game playing, modeling group instruction, and debates" were not feasible or were difficult to conduct in the online environment so were abandoned.

Besides choosing suitable tasks, task design was also about posing questions or providing the structure to engage the online learners, as one MTE mentioned that their team designed several protocols to facilitate participants' online conversations. To another extreme, some MTEs opted for minimal participant online interactions; they chose heavy individual learning tasks, such as reading reflection and other independent work with little interference from the facilitator. Overall, task design, either about the choice of task type or about the questions of the tasks, was the greatest area adjustment that MTEs made for their online delivery.

Approximately a quarter of the responses about adjustments concerned the *technological tools* used. Some MTEs utilized the technological environment to incorporate more mathematical investigation activities using technology, such as geometry explorations. Other MTEs incorporated multitudes of online resources such as applets, online videos, and simulations as teaching and learning resources for students. Strictly speaking, such adjustments were about activity design of the online course. Technological tool was a separate category from the task/activity design, because the decisions had an emphasis on adapting online resources and technologies for task design rather than restructuring tasks/activities for online delivery.

Besides tasks and tools, technology platforms were also mentioned as a concern for engaging participants in the online learning environment. Technology platforms or online platforms refer to the choice of synchronous or asynchronous environment. Synchronous environment include chat rooms, Adobe Connect, and Google Hangout, while synchronous environments may include the use of discussion forums, social media, and others. Either environment has its advantages and disadvantages.

Some MTEs felt that the choice of online platform was the key adjustment that they made to maximize teachers' learning, as one said, "I thought about … which environments to use in each year also to support teachers' discussion, collaboration, and dissemination of what they learned and created." In many cases, the platform was institutionally determined, the MTEs' choices were over which ones to use and in what combinations.

Other adjustment decisions concerned the instructional methods, content organization, or even class grouping structure. Four MTEs said that they had included more lecture style or presentation work in the online platform. Nine MTEs' important adjustments were about PD content organization: three of them had module structure; others had rigid structures where certain activities were organized strictly one following another (see examples in Table 3). Only one response concerned about the major adjustments were groupings (Table 3 last row).

The six themes on adjustments were identified. However, these six themes were intimately related. Adjustments on one aspect, such as choice of technological tools, was highly likely to cause adjustments on another aspect, such as task design or groupings of the participants. The total of the frequencies in Table 3 does not add up to 57 precisely because of the interrelated relationships among these themes. Many MTEs' responses touched on multiple types of adjustments. The interdependent adjustment decisions provide evidence to conceptualize decision making space for online mathematics teacher education.

Discussion

Decision Space for Designing Online Mathematics Teacher Education

Boyd (1993) outlined eight descriptive dimensions of decisions that are essential for distance learning instructional design. In Boyd's description, these eight dimensions are four pairs of corresponding dimensions, which include (a) agreed goals (i.e., the purposes of the course goal) and corresponding rules and mechanism that make sure goals are achieved; (b) subject matter (i.e., content) and corresponding metaphors and views that illustrate the content; (c) choices of real and virtual spaces and corresponding media used in the space; and (d) participants and corresponding sociostructure of the participants (Figure 2).

The findings from the survey reflect Boyd's (1993) decision dimensions as well as provide evidence to extend the decision dimensions to online mathematics teacher education. The report on the course/PD foci provided a range of goals and objectives that were present in the current online mathematics teacher education courses, which addressed the "what for" dimension as well as touched on the "what" dimensions. The response of PD audience, including preservice teachers, in-service teachers, and school administrators, locally and globally addressed the "who" dimension.

The findings on the difficulties that MTEs had when delivery was online and the adjustments that they made addressed the opposite dimensions of "what for?" "what?" and "who?" They are about instructional methods (that is, "how to control?"), task design (that is, "how to illustrate?"), technological platforms and tools to use (that is, "what media?" and "where?"), and social groupings (that is, "with whom?"). Based on the findings, the decision space was revised to reflect the unique design decisions in the context of online mathematics teacher education.

- The dimension of "what for?" is the course/PD goals and foci.
- The dimension of "how to control?" is the methods of instruction – lecturing, discussion-based, investigation, or any combination of these instructional methods.
- The dimension of "what?" is the objectives, including mathematics content area if applicable.
- The dimension of "how to illustrate?" is the task design.
- The dimension of "where?" is the choice of online platforms, such as synchronous communication platform and course management technology.
- The dimension of "what media?" is the choice of technological tools to use including video, interactive tools, and mathematics tools.
- The dimension of "who?" is the course/PD participants.
- The dimension of "with whom?" is about the ways of online interactions among participants.

Figure 6 depicts these dimensions in the context of designing online mathematics teacher education courses. It is interesting to note that the major decisions about providing online mathematics teacher education courses were not about determining the goals, objectives, or audience as shown in the upper half of the dimension, but in fact, the major decisions were about overcoming the constraints that the online environment imposed on achieving the learning goals, such as providing structure to facilitate online discussions, or decisions about new ways that the online environment might bring to mathematics teachers' learning, such as designing technology-based mathematics activities.

These decisions are represented in the lower half of the decision space and are all interrelated and interdependent on each other, as evident in the survey findings. As such, the rays starting from the center are all closely connected to indicate that one dimension influences another.

The current literature in online mathematics teacher education as reviewed earlier has addressed various dimensions of their online programs. The emphasis on the "with whom?" dimension, that is, online interactions among participants seems to be the mostly discussed in the research community (e.g., Kastberg et al., 2014; Larkin & Jamieson-Protor, 2015; Chieu & Herbst, 2016; Hjalmarson, 2017).

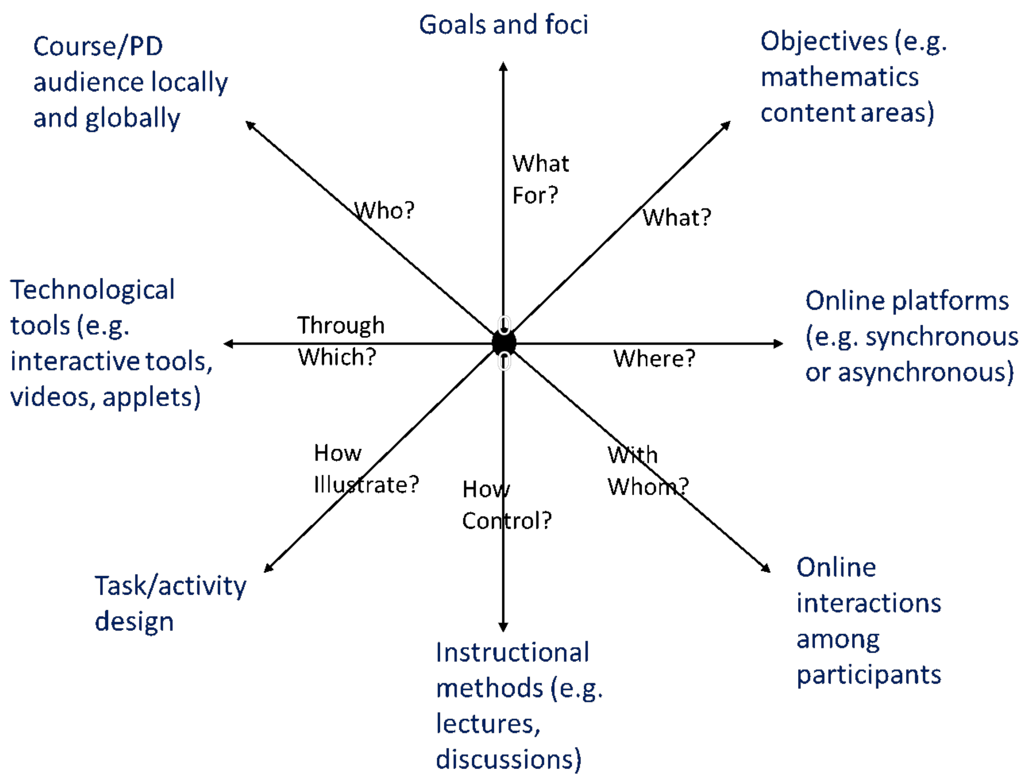


Figure 6. *Eight-dimension decision space for online mathematics teacher education.*

For Kastberg et al. (2014) and Hjalmarson (2017), the online communication placed a great challenge on their work with mathematics teachers, and the overcoming of this dimension brought effective results on their online programs. Chieu and Herbst (2016) put much effort in structuring social discussions in their online course, including establishing discussion norms and creating a series of animations of teaching for discussions. This approach involved other dimensions of decisions in Figure 8, such as technological tools, task design, and so forth, reflecting the connections of the eight dimensions.

Similarly, Larkin and Jamieson-Proctor (2015) utilized the online interaction dimension as a springboard to restructure the entire online course. Among these decisions, the hardest decisions or the most concerned decisions were related to task or activity design in the online space, as reported in the survey. However, current literature has limited studies discussing choice of tasks or designing tasks for online mathematics teacher education courses. Such findings inform the mathematics education community that further studies on the designing activity and designing tasks are needed.

Limitations of the Study

The survey intended to provide a status report of MTEs' practices in the US. In the survey, we asked participants to share only one online course or PD they had designed and implemented. However, many MTEs had designed multiple online courses or programs. One could assume that the survey participants might have reported on what they considered to be their most successful online course or the most recent one that they had implemented, which might not reflect their pedagogical practices of other online courses or programs that they implemented. To this end, the status report might not capture the overall MTEs' pedagogical practices, because they may have instructed other online courses and their practices might have been drastically different there.

Another limitation of this survey study is the lack of depth on some of the choices that the participants indicated. Both the theoretical and revised framework indicated that every dimension of decisions with regarded to the online course design was connected. Yet, the survey results could report only the individual dimensions that were important to the participants rather than report on the MTEs' one important dimension of decisions in relationship to other dimensions of decisions. More in-depth case studies will provide insights into these issues.

Implication for Practice

Results from the survey indicated that MTEs faced a number of difficulties when designing online teacher education courses. This study offers a framework to guide MTEs to support their online practices. The model from this study will help MTEs to identify the type of decisions they need to make. We reconceptualized Boyd's (1993) eight dimensions of decisions for the practice of online mathematics teacher education according to the survey results. In practice, one might further consolidate these eight dimensions into two groups of decisions.

The first group of decisions relates to identifying the course or PD goals, including identifying participants and needs of participants, and the second group of decisions relates to choices of content activities and online organization of content activities, including choices of online media and social grouping for content activities. These two groups of decisions could be regarded as two design stages. One stage of design informs the other. As MTEs design the online program, it is helpful to first consider the needs of the teachers' and set PD goals with references to the eight dimensions in Figure 8 and identify which dimensions are more imperative than others, as the survey results that prioritized decisions existed.

Further, the survey results raised potential discussions for the mathematics teacher education community on important issues that would support the practices of online mathematics teacher educations, such as designing activities and tasks that are authentic and effective to be delivered online. Such discussions need to be specific about the type of online courses where these tasks are used, since tasks used in mathematics content focused courses or sessions could be drastically varied in nonmathematics-content-focused courses or sessions.

In the survey results MTErs saw that implementing nonmathematics-content-focused courses online were less challenging than implementing mathematics content focuses courses online. In-depth discussions on how tasks or activities are adjusted for the online environment, in terms of their types, their pedagogical, mathematical, and technological elements, would be beneficial for furthering the work of online mathematics teacher education.

References

Anderson, T. (2008). *Theory and practice of online learning*. Athabasca, CAN: Athabasca University Press.

Anderson, T., Rourke, L., Garrison, R., & Archer, W. (2001). Assessing teaching presence in computer conferencing transcripts. *Journal of Asynchronous Learning Networks, 5*(2).

Boyd, G. (1993). A theory of distance education for the cyberspace era. In D. Keegan (Ed.), *Theoretical principles of distance education* (pp. 234-253). New York, NY: Routledge.

Chieu, V. M., & Herbst, P. (2016). A study of the quality of interaction among participants in online animation-based conversations about mathematics teaching. *Teaching and Teacher Education, 57*, 139-149. doi:10.1016/j.tate.2016.03.010

Clay, E., Silverman, J., & Fischer, D. (2012). Unpacking online asynchronous collaboration in mathematics teacher education. *ZDM, 44*(6), 761-773. doi:10.1007/s11858-012-0428-8

Corbin, J., & Strauss, A. (2007). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (3rd ed.). Thousand Oaks, CA: Sage Publications.

Crockett, C. C. (2010). *The impact of online professional development on the instructional practices of K-12 mathematics teachers* (Doctoral disseration). Available from ProQuest Dissertations and Theses A&I database. (UMI No. 3457967)

Dede, C., Jass Ketelhut, D., Whitehouse, P., Breit, L., & McCloskey, E. M. (2009). A research agenda for online teacher professional development. *Journal of Teacher Education, 60*(1), 8-19.

Engelbrecht, J., & Harding, A. (2005). Teaching undergraduate mathematics on the internet. *Educational Studies in Mathematics, 58*(2), 253-276. doi:10.1007/s10649-005-6457-2

Garrison, D. R., Anderson, T., & Archer, W. (2001). Critical thinking, cognitive presence, and computer conferencing in distance education. *American Journal of Distance Education, 15*(1), 7-23.

Garrison, D. R., Anderson, T., & Archer, W. (2010). The first decade of the community of inquiry framework: A retrospective. *The Internet and Higher Education, 13*(1), 5-9.

Gunawardena, C. N., & McIsaac, M. S. (2004). Distance education. In D. H. Jonassen (Ed.), *Handbook of research on educational communications and technology* (pp. 355-395). Mahwah, NJ: Erlbaum.

Hjalmarson, M. A. (2017). Learning to teach mathematics specialists in a synchronous online course: A self-study. *Journal of Mathematics Teacher Education, 20*(3), 281-301. doi:10.1007/s10857-015-9323-x

Holmberg, B. (1991). Testable theory based on discourse and empathy. *The Journal of Open and Distance Learning, 6*(2), 44-46.

Kastberg, S., Lynch-Davis, K., & D'Ambrosio, B. (2014). Examining mathematics teacher educators' emerging practices in online environments. *Contemporary Issues in Technology and Teacher Education, 14*(4), 384-400. Retrieved from https://www.citejournal.org/volume-14/issue-4-14/mathematics/examining-mathematics-teacher-educators-emerging-practices-in-online-environments

Keegan, D. (1993). *Theoretical principles of distance education*. New York, NY: Routledge.

Larkin, K., & Jamieson-Proctor, R. (2015). Using transactional distance theory to redesign an online mathematics education course for pre-service primary teachers. *Mathematics Teacher Education & Development*, *17*(1), 44-61.

Lehman, R. M., & Conceição, S. C. (2010). *Creating a sense of presence in online teaching: How to" be there" for distance learners*. San Francisco, CA: Jossey-Bass.

Moore, M. G. (1993). Theory of transactional distance. In D. Keegan (Ed.), *Theoretical principles of distance education* (pp. 22-38). New York, NY: Routledge.

Pape, S. J., Prosser, S. K., Griffin, C. C., Fichtman Dana, N., Algina, J., & Bae, J. (2015). Prime online: Developing grades 3-5 teachers' content knowledge for teaching mathematics in an online professional development program. *Contemporary Issues in Technology & Teacher Education, 15*(1), 14-43. Retrieved from https://www.citejournal.org/volume-15/issue-1-15/mathematics/prime-online-developing-grades-3-5-teachers-content-knowledge-for-teaching-mathematics-in-an-online-professional-development-program

Schwartz, C. (2012). Counting to 20: Online implementation of a face-to-face, elementary mathematics methods problem-solving activity. *TechTrends, 56*(1), 34-39. doi:10.1007/s11528-011-0551-3

Vertecchi, B. (1993). Structural analysis of distance education. In D. Keegan (Ed.), *Theoretical principles of distance education* (pp. 133-140). New York, NY: Routledge.

Whitehouse, P., Breit, L. A., McCloskey, E. M., Ketelhut, D., & Dede, C. (2006). An overview of current findings from empirical research on online teacher professional development. In C. Dede (Ed.), *Online professional development for teachers: Emerging models and methods* (pp. 13-29). Cambridge, MA: Harvard Education Press.

Appendix
Sample Questions

Survey Section II Sample Question

How often did you use the following approaches in delivering the online sessions? Please drag following items in the appropriate boxes.

1. Discussing skills and methods for computations
2. Discussing multiple representations and interpretations of central ideas
3. Solving complex or unfamiliar problems
4. Investigating technology-based problems
5. Connecting different mathematical ideas
6. Discussing specific mathematical thinking processes (e.g. proving)
7. Analyzing learners' mathematics
8. Designing tasks
9. Reviewing curriculum materials
10. Analyzing specific teaching practices (e.g. questioning)'
11. Discussing social justice issues
12. Discussing research in mathematics education
13. Please specify: ____________________

Response Options: "Never," "Occasionally," or "To a considerable degree."

Survey Section III Sample Question

What difficulties did you encounter when you designed/delivered the course online? Select all that apply.

1. Engaging participants from different backgrounds
2. Selecting appropriate online tools and/or mathematics specific technology
3. Selecting tasks that are compatible with the online environment or other technology used
4. Posing tasks or discussion prompts for desired online interaction
5. Organizing online grouping
6. Following and providing feedback to participants' posts
7. Getting participants to use online technology or other selected technology
8. Please specify: ____

Survey Section IV Sample Questions

In your opinion, which of the following courses/PD are easier to deliver online and which ones are easier to deliver face-to-face? Please drag following items in the appropriate box.

1. Mathematics content course/PD
2. Mathematics teaching methods course/PD
3. Mathematics educational theory course/PD
4. Technology integration course/PD
5. Mathematics curriculum or assessment course/PD
6. Interdisciplinary course/PD
7. Large scale mathematics teacher PD
8. Small scale mathematics teacher PD

9. Teachers of different grade levels and/or background
10. Teachers of similar grade levels and/or background

Response Options: "Easier to deliver online" or "Easier to deliver face-to-face."

Lastly, in your opinion, are there any mathematical topics that are more difficult to teach online than others? Please briefly provide reasons.